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ABSTRACT

This exploratory study investigates first-, second-, and third-grade children's performance on selected open addition and subtraction sentences presented in verbal and symbolic formats. For both the verbal and symbolic formats, three first-grade inventories and three upper-grade inventories were constructed. Sixteen children at each of the first-, second-, and third-grade levels were randomly selected to participate in this pilot investigation. Eight first-grade children responded to a one-digit symbolic inventory and eight subjects responded to a one-digit verbal inventory. Eight second-grade children and eight third-grade children responded to a two-digit symbolic inventory with the remaining second- and third-grade children responding to a two-digit verbal inventory. Based on the results of this pilot study, further research is recommended. (HM)

Children's Performance on Selected Addition and Subtraction Situations Involving the Existence and Non-existence of Whole Number Solutions: A Report of Initial Piloting

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Purpose

The purpose of this investigation is to examine first-, second-, and third-grade children's performance on selected open addition and subtraction sentences presented in verbal and symbolic formats. Specific aspects of the children's performance which this investigation will focus on will be discussed in detail as the rationale is developed.

Background and Rationale

Table 1 provides the reader with the item models considered in this investigation along with some specific examples of some of the items presented to the subjects.

Table 1

Symbolic Item Models

A1	A2	S1	S2
a + ___ = c	___ + b = c	a - b = ___	a - ___ = c

Examples

4 + ___ = 9	___ + 5 = 8	7 - 2 = ___	6 - ___ = 4
6 + ___ = 2	___ + 7 = 5	6 - 9 = ___	3 - ___ = 7

Verbal Format Examples

A1: Susan has 4 pennies. Her mother gave her some more pennies. Now Susan has 7 pennies. How many pennies did her mother give Susan?

Note: Similar verbal examples can be constructed from the other three item models.

Several observations should be made at this point concerning the information presented in Table 1. To begin with, all of the examples *can* be solved using the subtraction operation or a subtractive process such as counting back. This does not mean that a subject will employ a solution method of this type, but simply could do so if desired. Some of the data collected will relate to the solution method employed on the various items presented to a subject. As can also be noted, some of the examples appearing in Table 1 were constructed so that no whole number solution existed. Examples of the remaining two addition and subtraction item models, $a + b = \_\_$ and $\_\_ - b = c$, cannot be constructed so that a whole number solution does not exist.

Children's performance on items similar to those in Table 1 is of interest for several reasons. The decade of the 1970's has meant some profound changes in the thinking of some mathematics educators as the back-to-basics movement and the minimal competency movement have increased their influence. One of the differences in the elementary school mathematics curriculum of the 1970's and the 1960's is the popularity of the missing addend models, $a + \_\_ = c$ and $\_\_ + b = c$. In the modern math era, these models were popularly seen as a means of introducing the operation of subtraction. The question should then be raised as to whether children see these items as ones involving subtraction. If children must be trained to see the relationship existing with subtraction, perhaps there is a better way to introduce subtraction. The first- and second-grade children to be interviewed in this investigation will have had little instructional contact with all examples except for ones such as $7 - 2 = \_\_$.

The second aspect of this investigation which will receive emphasis is children's performance on those items constructed so that no whole number solution exists. Typically, when the operation of subtraction is first introduced, children are simply and rotely told that one "always subtracts the smaller number from the larger number." Considering the absence of any meaningful instruction, it is not surprising that children forget this statement and invent their own procedures to handle test items, or generalize that these items are solved in the same way as similar items having a whole number solution. One of the questions raised in this investigation is whether young children are aware of the importance of the order of the numbers appearing in a subtraction example. In general, if a child treats an example such as 6 - 9 = __ in the same way s/he treats the commuted version of this example, then the child will be judged to be unaware of the importance of the order of the numbers. Please note that this is not the same as stating that the child believes subtraction to be a commutative operation. Based on an investigation of children's ability to detect items with no whole number solution, Weaver (1972) conjectured that many children may "overgeneralize to subtraction their knowledge of the commutativity of addition." Although the investigation reported in this manuscript will not support or refute Weaver's conjecture, it will take an intermediate step in determining children's awareness of the importance of order in a canonical subtraction open sentence.

Research Design and Procedure

The subjects interviewed in this pilot investigation were 16 children at each of the first-, second- and third-grade levels. Eight first-grade children responded to an inventory similar to the one in Appendix 1 and eight subjects responded to a verbal inventory similar to the one in Appendix 2. Eight second-grade children and eight third-grade children responded to a symbolic inventory similar to the one in Appendix 3 with the remaining second- and third-grade children responding to a verbal inventory similar to the one in Appendix 4. All children interviewed in this investigation were randomly selected from all first-, second- and third-grade children who returned parental permission slips. The total number of children attending the school in question is approximately 140.

The Inventories

For both the verbal and symbolic formats, three first-grade inventories and three upper-grade inventories were constructed. For each inventory, new items were selected from designated pools with the order of the items randomly rearranged for each inventory. The only exception was that the first item on each inventory was required to have a whole number solution.

The one-digit item pool consisted of basic addition facts having a sum between five and 11 and non-zero addends. $2 + 7 = 9$ is one example while $2 + 3 = 5$ and $6 + 5 = 11$ are non-examples. After an item was selected, it was modified to fit the model in question using established procedures and the desirability or non-desirability of having a whole number solution. Hence, $2 + 7 = 9$ could be modified to become $9 - \_\_ = 2$, if desired.

The two-digit item pool consisted of examples modeled by $a + b = c$ where a, b, and c represent two-digit numbers. Furthermore, items such as $29 + 14 = 43$, in which regrouping might be necessary were excluded. Hence, only items such as $23 + 22 = 45$ were included. These items were then modified in the same way as the one-digit items.

The Interviews

Each child was individually interviewed. The child's name and teacher were obtained and then the child was introduced to an 'Oscar the Grouch' doll and invited to help Oscar work some number puzzles. The child then worked through a simple addition puzzle in either symbolic or verbal context. For example, if a symbolic inventory was to be administered, the example $3 + 5 = \blacksquare$ was presented. The task was to first determine if a number was hiding behind the black box, and if so, to determine which one it was. In the initial directions, the child was warned that some number puzzles might be new. The child was then told if s/he believed that no number was hiding behind the black box, or if s/he could not solve the puzzle, to simply say so. This latter statement was re-read to the child after every fourth item.

Findings

Any statement concerning the findings or results of this investigation must be tempered with the reminder that this study was exploratory in nature and employed a small sample size. However, some data will be presented not for the purposes of making a conclusive argument, but to encourage further work in this area. Some research questions will now be stated along with the preliminary indications as to how these questions will ultimately be answered. Q1: The teachers of the children interviewed used a mathematics series which did not begin instruction on the missing addend models until grade three. The teachers stated that it was their belief that children would readily learn how to handle these items if the children received a solid foundation in straightforward addition and subtraction situations, i.e., $a + b = \_\_$ and $a - b = \_\_$. Given the lack of instruction in this area, how successful were children in solving the missing addend and missing subtrahend items?

Table 2 contains the raw data relating to children's success on inventory items. If the reader focuses on the cells relating to the one-digit symbolic items A1, A2 and S2 (those missing addend or subtrahend items with a whole number solution), it can be seen that for all grade levels, at least 75% of the children reponded correctly.

Table 2

Grade One

Item Models (One-digit Number Items)

	A1	A1N*	A2	A2N*	S1	S1N*	S2	S2N*
Sym	6	7	6	7	8	4	8	7
Ver	4	2	3	2	6	2	6	2

The number in a given cell indicates the number of children who responded correctly (out of 8 possible) to a given item.

Grade Two

Item Models

	A1	A1N	A2	A2N	S1	S1N	S2	S2N	A1	A1N	A2	A2N	S1	S1N	S2	S2N
Syn	7	6	7	6	8	3	6	6	1	7	0	7	4	5	2	6
Ver	6	2	7	1	8	4	8	4	3	2	1	1	5	3	3	3

Grade Three

Item Models

	A1	A1N	A2	A2N	S1	S1N	S2	S2N	A1	A1N	A2	A2N	S1	S1N	S2	S2N
Sym	8	7	8	6	8	6	7	5	1	5	-3	6	6	4	5	6
Ver	7	6	8	7	8	7	8	7	7	6	5	6	6	6	7	6

*In all cases, 'N' refers to a no whole number solution item.

For the two-digit symbolic items, the situation is different. The children were relatively successful only on the missing subtrahend items. One possible explanation for the lack of success was that children most frequently attempted to solve these items mentally.

For the verbal items A1, A2, and S2, first-grade children were successful approximately 50% of the time. For the items A1 and A2, children did not transform equations and use a subtractive process. Perhaps some instruction in this area would improve results. However, on all verbal items, children made modeling errors. Hence, if better results are desired, perhaps a simpler solution would be to give children more contact with verbal items and some instruction concerning modeling. However, by grade two, children were successful in responding to one-digit items.

Q2: This question concerns children's awareness of the importance of the order of the numbers in a straightforward subtraction item. For this investigation, a child was coded to be unaware of the importance of order of the numbers in a subtraction sentence, if the child gave one of the following justifications for his/her response to an item such as $6 - 9 = \underline{3}$.

(1) 'It's the same as $9 - 6 = \_\_$.'

(2) 'Because $3 + 6 = 9$.'

(3) If the child quickly and seemingly rotely recalled '3'.

A child whose response was '0' or '15' (for the item above) was judged to be aware that order mattered. As can be seen in Table 2, many children responded by stating that no whole number was hiding behind the black box which is perhaps the best that can be expected from young children. For this reason, this response is coded as being 'correct'. The research question is simply whether children are aware of the importance of the order of the numbers in a subtraction sentence.

The data pertaining to this question is presented in Table 3. If these findings hold in a more extensive investigation, I believe the conclusion should be that the situation is unsatisfactory. Unawareness of the importance of order may be for the child, a step in the direction towards generalizing that subtraction is a commutative operation. Further research in this area is necessary. For the classroom teacher, most likely it is the case that instruction concerning the commutativity of addition, or the order of the numbers in an addition sentence, should be accompanied by instruction concerning the different situation in subtraction.

Table 3

Awareness of the Importance of Order
in a Canonical Subtraction Sentence:

Grade Level		
One	Two	Three
2	5	3

The figures represent the number of children at each level who seemed unaware of the importance of order.

Justifications: Given $4 - 6 = ■$: Q: How do you know that '2' is hiding behind the black box?

Two S's: Because $2 + 4 = 6$
Two S's: Because $4 - 6 = 6 - 4$
Six S's: Because $4 - 6 = 2$

Based on this and previous piloting, the areas focused on in this paper merit future empirical attention.

References

Weaver, J. Fred The ability of first-, second-, and third grade pupils to identify open addition and subtraction sentences for which no solution exists within the set of whole numbers. School Science and Mathematics, 1972, 72, 679-691.

Appendix 1

A One-digit Symbolic Inventory

A1:	2 + ___ = 6	A1N:	9 + ___ = 5
A2:	___ + 4 = 8	A2N:	___ + 9 = 3
S1:	10 - 1 = ___	S1N:	3 - 9 = ___
S2:	10 - ___ = 6	S2N:	5 - ___ = 9

Appendix 2

A One-digit Verbal Inventory

A1: Susan has 4 pennies. Her mother gave her some more pennies and now she has 10 pennies. How many pennies did her mother give her?

A1N: Bob has 9 books. His teacher gave him some more books and now he has 2 books. How many books did Bob's teacher give him?

A2: There are some cookies in the jar. Sara put 3 more cookies in the jar and now there are 8 cookies in the jar. How many cookies were in the jar to start with?

A2N: Some people are on the train. 10 more people got on the train and now there are 7 people on the train. How many people were on the train to start with?

S2: Bill has 9 marbles. He gave some marbles to Jill and now he has 7 marbles. How many marbles did he give to Jill?

S2N: Jan has 1 book. She gave some books to Tony and now she has 8 books. How many books did she give to Tony?

S1: Sally had 10 frogs in a box. 9 frogs jumped out of the box. How many frogs does Sally have left in the box?

S1N: Leroy had 4 pieces of candy. He gave 6 pieces to Jenny. How many pieces of candy does he have left.

Appendix 3

A Two-digit Symbolic Inventory*

A1:	22 + ___ = 43	A1N:	58 + ___ = 34
A2:	___ + 41 = 62	A2N:	___ + 38 = 17
S1:	62 - 41 = ___	S1N:	47 - 69 = ___
S2:	49 - ___ = 27	S2N:	32 - ___ = 56

*Note: Second- and third-grade children also responded to a one-digit inventory and thus, responded to 16 items.

Appendix 4

A Two-digit Verbal Inventory*

A1: Joe has 57 stamps. He bought some more stamps. Now Joe has 69 stamps. How many stamps did Joe buy?

A1N: There are 65 people on the airplane. Some more people got on. Now there are 53 people on the airplane. How many people got on?

A2: Some books are on a shelf. Bess put 63 more books on the shelf. Now there are 87 books on the shelf. How many books were on the shelf to start with?

A2N: Some children were ice-skating. 53 more children joined them. Now there are 31 children ice-skating. How many children were ice-skating to start with?

S1: There are 29 cans of paint in the store. 13 cans were sold. How many cans of paint are left in the store?

S1N: Wally has 36 pennies. He spent 58 pennies to buy some candy. How many pennies does Wally have left?

S2: There are 67 shells in a box. Some shells were taken out. Now there are 52 shells in the box. How many shells were taken out of the box?

S2N: There are 42 people on the bus. Some people got off. Now there are 55 people on the bus. How many people got off the bus?

*Again, children responded to a one-digit verbal inventory.